Theoretical Basis for Rosgen's Pagosa Good/Fair Equation

D.D. Hinton & R.H. Hotchkiss
Brigham Young University, Provo, Utah, USA

T.P. Ballestero
University of New Hampshire, Durham, New Hampshire, USA

ABSTRACT: There is a school of thought regarding the prediction of bedload transport which relies on prolific amounts of data fitted by regression curves that capture the physical processes of bedload transport. A compelling argument against this school of thought is that the use of regression curves do not account for or explain the physics known to be driving bedload transport. This work addresses the apparent lack of a theoretical basis to the Rosgen Pagosa Curve.

Beginning with the Parker Surface-based Bedload Equation, we derive the Rosgen Pagosa Curve for good/fair streams. Then, this work demonstrates the derived expression closely approximates the Pagosa curve and validates the expression using data from five different rivers. The proposed expression illustrates that, if the basic variables involved in the phenomenon are employed in regressions, the resulting equations embrace the theoretical treatments and the exponents and coefficients serve to improve the description of natural variability.

1 INTRODUCTION

Efforts to understand bedload transport typically fall into two schools of thought. The first uses mathematical relationships from known physics-based processes related to bedload transport. The second uses an empirical approach where data is collected and synthesized into regression equations which can then be applied to other similar streams.

The second, or regression curve, school has been criticized for ignoring the physics known to be initiating bedload transport [*Montgomery & MacDonald* 2002, *Simon et al.* 2007]. It can be argued, however, that the physics have been captured by the data and are thereby reflected in the regression relation. It is feasible then to cast the fitted curve in a form that resembles the physics-based equations and demonstrates the theoretical processes represented in the sampled data from which it was created. Since both approaches have physical relationships in their essence, it is expected that there should be similarity in the final forms of the two. By casting the regression curve into a format that accounts for physics-based processes and displays the curve's theoretical basis, it is hoped that greater collaboration could exist between the two schools of thought.

2 EQUATION COMPARISON

2.1 *Types of Bedload Prediction Methods*

Most bedload prediction equations developed from known physics processes can be organized into two basic categories. The first is the rating curve and the second is the incipient motion method.

The rating curve methods establish a power law relationship between discharge and transport rate. The coefficient and exponent are site specific. While methods for developing predictive curves for universal application exist, most rating curves are empirically derived using bedload measurements from the site in question.

On the other hand, incipient motion equations develop a ratio between shear stress and reference shear stress for a given flow scenario. This ratio represents the transport stage and describes whether sediment on the river or stream bed surface is likely to move [*Pitlick et al.* 2009]. The threshold of transport stage for which sediment of any given size is prone to move is referred to as incipient motion [*Wilcock et al.* 2009]. The transport stage is then used to create a dimensionless bedload parameter W*. Recognizing that countless transport stages are possible for any given stream, W* collapses the transport stage relationship into a single, piecewise equation that applies universally to all flow conditions of the given stream. While various methods exist for deriving W* [*Crowe* 2003, *Parker* 1990, *Parker & Klingeman* 1982, *Wilcock* 2001], the end result is a bedload prediction equation of essentially the same format as shown in Equation 1.

$$Q_b = \frac{W^* T u_*^3}{Rg} \tag{1}$$

where Q_b is the bedload transport rate [M/T], T is top width [L], u_* is shear velocity [L/T], R is the submerged density equal to $\rho_s/\rho - 1$, and g is the gravitation coefficient [M/T^2] [*Pitlick et al.* 2009].

2.2 *Pagosa Curve*

The Pagosa Curve developed by David Rosgen [*Rosgen et al.* 2006] does not strictly fall within either of the two categories listed previously. Although a rating curve, the Pagosa curve is dimensionless like the transport stage method. Unlike the transport stage method, however, the Pagosa Curve is non-dimensionalized using values of discharge and transport rate at bankfull flow.

Created using data from a series of Southwest Colorado streams, the Pagosa Curve method requires streams to be delineated according to a stability index. If a stream preserves its dimensions and general shape over time without degradation and aggradation it is assigned a good/fair rating; otherwise the stream is given a poor stability rating. A separate dimensionless bedload curve exists for both the good/fair and poor scenarios [*Rosgen et al.* 2006].

The controversy surrounding the Pagosa Curve is due largely to its non-traditional format and unconventional methods. Instead of using a fluid mechanics approach by using a form of incipient motion as the reference condition, the Pagosa dimensionless parameter takes a geomorphic approach and uses bankfull as a reference. The selection of the bankfull discharge is also somewhat controversial in that it is hard to identify even in the stream and there are disagreements regarding the frequency of bankfull discharge [*Doyle et al.* 2007]. An additional concern voiced by critics focus on the use of single exponent to represent all flow conditions.

The concerns regarding the Pagosa Curve exponent are partially mitigated by comparing it with other proposed rating curves. One such rating curve is a general power equation proposed by Barry et al [*Barry et al.* 2004]. The exponent of the equation is calculated using site specific parameters and varies from site to site. However, at any given site the exponent is the same for the full range of discharges. Exponents calculated for over 20 streams in Idaho ranged from 1.5 to 4 [*Barry et al.* 2004]. This range encompasses the value used in the Pagosa Curve.

Many of the other concerns voiced by critics of the Pagosa Curve are due to its non-traditional format as a dimensionless rating curve. This concern can be mitigated by casting the Pagosa Curve in a format similar to the transport stage methods.

3 PAGOSA CURVE TRANSFORMATION

3.1 *Exponent*

Shear velocity and the Manning Equation can be written as

$$u_* = \sqrt{g R_h S} \tag{2}$$

$$Q = \frac{\mathrm{k}}{n} A R_h^{\frac{2}{3}} \sqrt{S} \tag{3}$$

where Q is discharge [L^3/T], k is a coefficient equal to 1.0 for S.I. units (1.49 for English units), n is the roughness coefficient, A is the cross-sectional flow area [L^2], R_h is the hydraulic radius [L], and S is the slope [L/L]. Rearranging Equation 3 to solve for S and then substituting into Equation 2 gives Equation 4.

$$u_* = \frac{Qn\sqrt{g}}{kAR_h^{1/6}} \tag{4}$$

Replacing shear velocity in Equation 1 with Equation 4 and then simplifying results in Equation 5.

$$Q_b = \frac{W^* T n^3 \sqrt{g}}{R k^3 A^3 \sqrt{R_h}} Q^3 \tag{5}$$

This process can be repeated for any given flow. Performing this process for bankfull conditions provides opportunity to develop a ratio of an arbitrary flow rate with that of bankfull. This ratio, G*, can be simplified as

$$G^* = \frac{W^* T A_{bf}^3 \sqrt{R_{h_{bf}}}}{W_{bf}^* T_{bf} A^3 \sqrt{R_h}} Q_*^3 \tag{6}$$

where the subscripts bf refer to bankfull conditions and Q_* refers to dimensionless discharge derived by the ratio of the given discharge (Q) with bankfull discharge (Q_{bf}). Assuming a rectangular channel, A^3 can be separated into (TH)A^2. Separating thus for A and A_{bf} produces Equation 7.

$$G^* = \frac{W^* T T_{bf} H_{bf} A_{bf}^2 \sqrt{R_{h_{bf}}}}{W_{bf}^* T_{bf} T H A^2 \sqrt{R_h}} Q_*^3 \tag{7}$$

Hydraulic relationships proposed by Parker [1979] for wide channels provide the next piece of the puzzle. Parker's relationship for slope is

$$S = \frac{0.0662 B_*^{0.819}}{\breve{Q}^{0.819}} \tag{8}$$

where

$$B_* = \frac{T}{D_{50}} \tag{9}$$

$$\breve{Q}_* = \frac{Q}{\sqrt{RgD_{50}}(D_{50})^2} \quad (10)$$

where D_{50} is the median grain size particle. Substituting Equations 9 and 10 into Equation 8 and rearranging results in Equation 11.

$$\left(\frac{T}{D_{50}}\right)^{0.819} = \frac{SQ^{0.819}}{0.0662\left[\sqrt{RgD_{50}}D_{50}{}^2\right]^{0.819}} \quad (11)$$

Multiplying both sides by $T^{0.181}(D_{50})^{0.819}$ and then simplifying gives Equation 12.

$$T = \frac{SQ^{0.819}T^{0.181}}{0.0662D_{50}^{1.229}\left(\sqrt{Rg}\right)^{0.815}} \quad (12)$$

Repeating for bankfull top width (T_{bf}), substituting T and T_{bf} into Equation 7, and then simplifying gives Equation 13.

$$G^* = \frac{W^*TQ_{bf}^{0.819}T_{bf}^{0.181}H_{bf}A_{bf}^2\sqrt{R_{h_{bf}}}}{W_{bf}^*T_{bf}Q^{0.819}T^{0.181}HA^2\sqrt{R_h}}Q_*^3 \quad (13)$$

Assuming a wide channel such that Rh is roughly equivalent to the flow depth, Equation 14 is derived.

$$G^* = \frac{W^*T^{0.819}Q_{bf}^{0.819}H_{bf}^{3/2}A_{bf}^2}{W_{bf}^*T_{bf}^{0.819}Q^{0.819}HA^2}Q_*^3 \quad (14)$$

Recognizing that

$$\frac{Q_{bf}^{0.819}}{Q^{0.819}} = Q_*^{-0.819} \quad (15)$$

Equation 16 can thereby be developed.

$$G^* = \frac{W^*T^{0.819}H_{bf}^{3/2}A_{bf}^2}{W_{bf}^*T_{bf}^{0.819}HA^2}Q_*^{2.181} \quad (16)$$

For comparison, the good/fair version of the Pagosa regression equation is

$$G^* = -0.0113 + 1.0139Q_*^{2.1929} \quad (17)$$

where G^* and Q_* were defined previously [*Rosgen et al.* 2006]. Neglecting the intercept (which likely relates to incipient motion), the primary focus is the coefficient and exponent. The exponent derived here of 2.181 is strikingly similar to the value of 2.19 used in Equation 17.

The coefficient of Equation 17 is independent of channel geometry and flow conditions unlike Equation 16. In order for Equations 16 and 17 to be compatible, the average coefficient of any given stream over a range of discharges must approximate unity.

3.2 *Coefficient*

To test how closely the coefficient of Equation 17 matches Equation 16, five gravel-bed streams were selected to determine an average coefficient. Selection criteria for the five sites included availability of the necessary data and a stream width to depth ratio at bankfull greater than 10 so that the wide channel assumption could prevail. Some of the necessary data include channel geometry, bankfull discharge, bankfull channel geometry, and measurements of discharge and bedload transport rate over a range of flow conditions. Some of the basic information are included in Table 1.

Table 1. Study sites and characteristics

River name	Basin A km^2	Slope m/m	$^*D_{50}$ mm	Q_{bf} m^3/s	**W/D
East Fork, WY	466	0.0007	5	20.0	15.0
Sagehen Cr, CA	27	0.0102	58	2.0	11.8
Big Wood, ID	356	0.0091	150	21.9***	14.4
Cache Cr., WY	28	0.0210	46	2.1	13.0
St. Louis Cr. #3	54	0.0190	82	4.6***	25.8

* Median diameter of the channel bed surface
** Width to depth ratio
*** Return period of 1.5 years

The study sites used in this analysis have been described by others. East Fork River, WY was sampled using a belt sampler as described in Leopold and Emmett [*Leopold & Emmett* 1976]. Sagehen Creek [*Andrews* 1994], Big Wood River [*King & Station* 2004], Cache Creek [*Ryan et al.* 2005], and St. Louis Creek [*Ryan et al.* 2002] were sampled using pressure differential samplers such as the Helley-Smith sampler.

The coefficient for Equation 16 was calculated for more than 300 individual measurements from the five sites listed in Table 1. The mean coefficient for each individual site was derived, followed by the overall mean coefficient for all 300 measurements.

3.3 *Results*

The average coefficient for each site is listed in Table 2.

Table 2. Overage coefficient for each site.

River name	Average Coefficient
East Fork, WY	1.9241
Sagehen Cr, CA	0.9844
Big Wood, ID	1.1569
Cache Cr., WY	1.0572
St. Louis Cr. #3	0.4307

The overall average coefficient derived from Equation 16 was 1.0128 compared to the value of 1.0139 of Equation 17. The difference between the derived value and the original coefficient (Equation 17) is a mere 0.11 percent.

3.4 *Discussion*

Of the five sites used in this analysis, East Fork and St. Louis Creek #3 had the greatest deviation from the coefficient proposed in Equation 16. Because the derivation of the exponent assumed the site exhibited good/fair stability characteristics, streams with a poor stability rating would not be expected to fit

Equation 16 as derived herein. It is therefore possible that East Fork and St. Louis Creek #3 did not exhibit good/fair stability characteristics.

Another potential reason for the deviation of East Fork River was the high amount of sand introduced upstream by irrigation return flows as observed by Lisle [*Lisle* 1995]. In his own analysis, Lisle considered East Fork River an outlier.

4 CONCLUSION

The analysis summarized in this paper bridges the gap between two schools of thought. By following the procedure described previously, the regression curve school is shown to be compatible with the format of the physics-based school of thought. The Pagosa Good/Fair Curve can be cast in the same format as the incipient motion method of bedload prediction.

The intent of this analysis is to draw the two schools of thought together. Schisms and divisions amongst those attempting to understand and master the phenomenon of bedload transport are detrimental and counterproductive. By minimizing the differences between the two schools, greater collaboration and synergy could exist which, in turn, would result in a better understanding of bedload transport.

REFERENCES

Andrews, E.1994. Marginal bed load transport in a gravel bed stream, Sagehen Creek, California. *Water Resources Research*. *30*(7). 2241-2250.

Barry, J.J., Buffington, J.M. & King, J.G.2004. A general power equation for predicting bed load transport rates in gravel bed rivers. *Water Resources Research*. *40*(10). W10401.

Crowe, J.C.2003. Surface-based transport model for mixed-size sediment. *Journal of Hydraulic Engineering*. *129*. 120.

Doyle, M.W., Shields, D., Boyd, K.F., Skidmore, P.B. & Dominick, D.W.2007. Channel-forming discharge selection in river restoration design. *Journal of Hydraulic Engineering*. *133*. 831.

King, J.G. & Station, R.M.R. 2004, *Sediment transport data and related information for selected coarse-bed streams and rivers in Idaho*. US Dept. of Agriculture, Forest Service, Rocky Mountain Research Station.

Leopold, L.B. & Emmett, W.W.1976. Bedload measurements, East Fork River, Wyoming. *Proceedings of the National Academy of Sciences*. *73*(4). 1000.

Lisle, T.E.1995. Particle size variations between bed load and bed material in natural gravel bed channels. *Water Resources Research*. *31*(4). 1107-1118.

Montgomery, D.R. & MacDonald, L.H.2002. Diagnostic Approach to Stream Channel Assessment and Monitoring. *JAWRA Journal of the American Water Resources Association*. *38*(1). 1-16.

Parker, G.1979. Hydraulic geometry of active gravel rivers. *Journal of the Hydraulics Division*. *105*(9). 1185-1201.

Parker, G.1990. Surface-based bedload transport relation for gravel rivers. *Journal of Hydraulic Research JHYRAF*. *28*(4).

Parker, G. & Klingeman, P.C.1982. On why gravel bed streams are paved. *Water Resources Research*. *18*(5). 1409-1423.

Pitlick, J., Cui, Y., Wilcock, P.R. & Station, R.M.R. 2009, *Manual for computing bed load transport using BAGS (Bedload Assessment for Gravel-bed Streams) Software*. United States Dept. of Agriculture, Forest Service, Rocky Mountain Research Station.

Rosgen, D.L., Silvey, H.L. & Frantila, D. 2006, *Watershed assessment of river stability and sediment supply (WARSSS)*. Wildland Hydrology.

Ryan, S.E., Porth, L.S. & Troendle, C.2002. Defining phases of bedload transport using piecewise regression. *Earth Surface Processes and Landforms*. *27*(9). 971-990.

Ryan, S.E., Porth, L.S. & Troendle, C.2005. Coarse sediment transport in mountain streams in Colorado and Wyoming, USA. *Earth Surface Processes and Landforms*. *30*(3). 269-288.

Simon, A., Doyle, M., Kondolf, M., Shields Jr, F., Rhoads, B. & McPhillips, M.2007. Critical Evaluation of How the Rosgen Classification and Associated “Natural Channel Design” Methods Fail to Integrate and Quantify Fluvial Processes and Channel Response1. *JAWRA Journal of the American Water Resources Association*. *43*(5). 1117-1131.

Wilcock, P.R.2001. Toward a practical method for estimating sedimentâ€transport rates in gravelâ€bed rivers. *Earth Surface Processes and Landforms*. *26*(13). 1395-1408.

Wilcock, P.R., Pitlick, J., Cui, Y. & Station, R.M.R. 2009, *Sediment transport primer: estimating bed-material transport in gravel-bed rivers*. US Dept. of Agriculture, Forest Service, Rocky Mountain Research Station.